Voting: You Can't Have Privacy without Individual Verifiability

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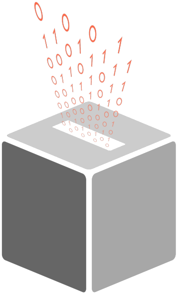
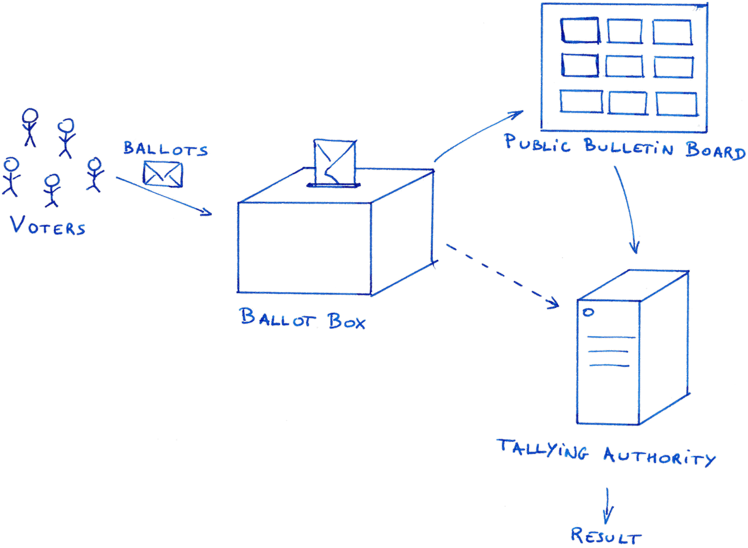
March 1, 2019

Introduction: e-voting protocols

- Using computers to organise elections
  $\longrightarrow$ voting machines in polling stations
  $\longrightarrow$ remote voting on the Internet
- More convenient
  $\longrightarrow$ for voters: vote from home, or abroad
  $\longrightarrow$ for authorities: easier to record and tally votes
- Many protocols have been proposed:
  Helios, Belenios, Civitas, Prêt-à-Voter,...
- But of course:
  need to ensure voting protocols are secure



Voting protocols



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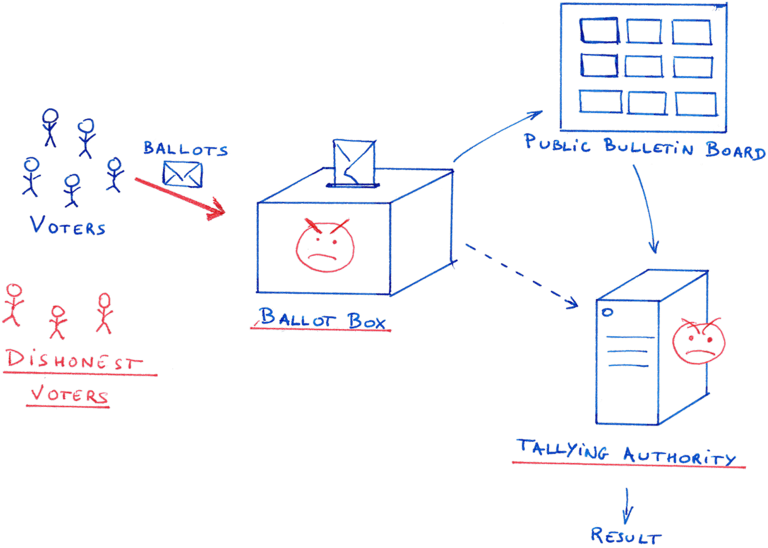

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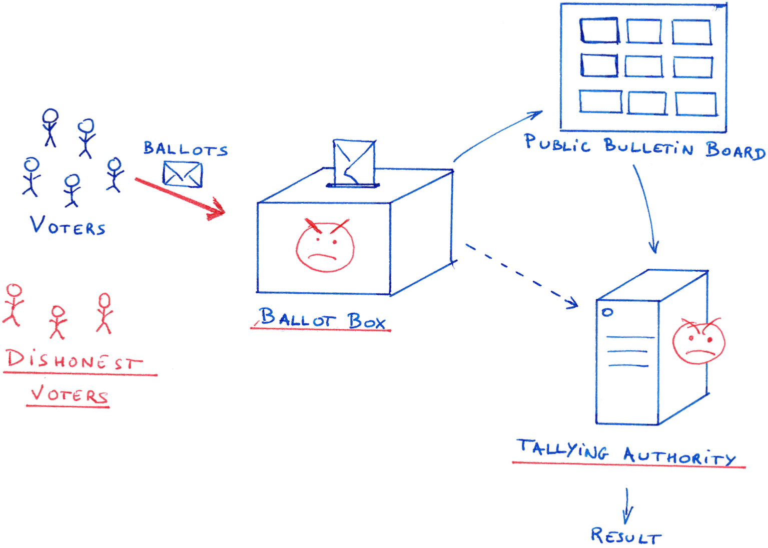


$\Longrightarrow$ What does it mean for a voting protocol to be secure?

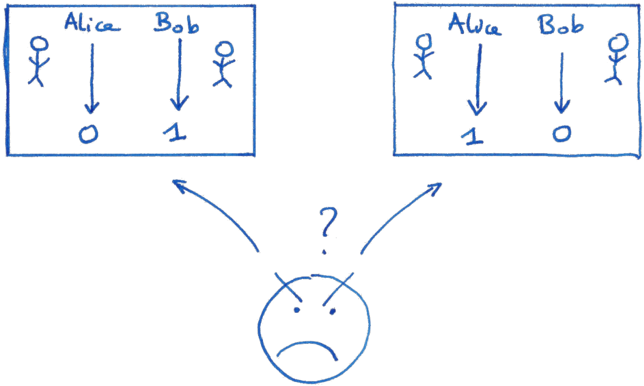
E-voting: security properties

Several properties have been defined:

- *privacy:*
  no one should know who I voted for
- *verifiability:*
  everyone can ensure that the votes are correctly counted
- *receipt-freeness/coercion resistance:*
  even if I want to, I can't prove who I voted for to someone else
- ...

Vote privacy

- What does it mean for the vote to be private ?
- An attacker is unable to tell *who voted for who*
- Indistinguishability property



Verifiability

Divided into three subproperties:

- *individual verifiability:*
  I can check that my vote is in the ballot box
- *universal verifiability:*
  everyone can check that the result corresponds to the ballot box
- *eligibility verifiability:*
  every ballot in the box was cast by a legitimate, registered voter

Privacy vs Verifiability

The two properties seem opposed:

- Privacy: give *no information* about how people voted
- Verifiability: give *enough information* to check each vote is counted

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  unconditional privacy and verifiability are incompatible

  (*i.e.* for an attacker with unbounded computing power)

- Regulations choose one over the other
  Ex: in France or Switzerland, privacy is prioritised over verifiability

Our result

Theorem (informal)

We show that, in fact,

$$\textit{Privacy} \Longrightarrow \textit{Individual Verifiability}$$

- Counter-intuitive, but does not contradict previous impossibility result $\longrightarrow$ our result is for a polynomial attacker
- How is it possible that some protocols are known to be private and non verifiable?
- What does this tell us about privacy?

Computational model

Voting scheme:

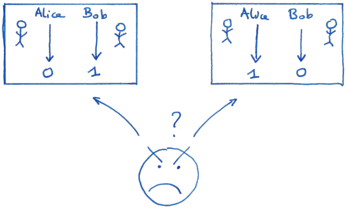
$$(\mathsf{Setup}, \mathsf{Vote}, \mathsf{VerifVoter}, \mathsf{Tally}, \mathsf{Valid})$$

- $\mathsf{Setup}(1^\lambda)$: generate the *election keys* $(\mathsf{pk}, \mathsf{sk})$
- $\mathsf{Vote}(id, \mathsf{pk}, v)$: construct a ballot containing the vote v for voter id
- $\mathsf{VerifVoter}(id, \mathsf{L}, \mathsf{BB})$: voter id checks her vote is counted in BB
- $\mathsf{Tally}(\mathsf{BB}, \mathsf{sk})$: compute the tally of the ballots on the board BB
- $\mathsf{Valid}(id, b, \mathsf{BB}, \mathsf{pk})$: checks that a ballot b cast by id is valid *w.r.t.* BB

counting function ρ: votes $\rightarrow$ result
with *partial tallying*: $\forall A, B.\ \rho(A \uplus B) = \rho(A) * \rho(B)$
Ex: multiset, sum, ...

Privacy: game-based definition

Privacy is defined as a cryptographic game
[Benaloh, 1987]



$\underline{\mathrm{Exp}_{\mathcal{A}}^{\mathrm{priv},\beta}(\lambda)}$

$(\mathrm{pk}, \mathrm{sk}) \leftarrow \mathrm{Setup}(1^\lambda)$
$\mathcal{A}_1^{\mathcal{O}_{\mathrm{vote}}^{\beta}, \mathcal{O}_{\mathrm{cast}}}(\mathrm{pk})$
if $\rho(\mathrm{V}_0) = \rho(\mathrm{V}_1)$ **then**
  $r \leftarrow \mathrm{Tally}(\mathrm{BB}, \mathrm{sk})$
  return $\mathcal{A}_2(\mathrm{pk}, r)$

$\underline{\mathcal{O}_{\mathrm{vote}}^{\beta}(id, v_0, v_1)}$

$b \leftarrow \mathrm{Vote}(id, \mathrm{pk}, v_\beta)$
$\mathrm{BB} \leftarrow \mathrm{BB} \| b$
$\mathrm{V}_0 \leftarrow \mathrm{V}_0 \| v_0$
$\mathrm{V}_1 \leftarrow \mathrm{V}_1 \| v_1$
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$\underline{\mathcal{O}_{\mathrm{cast}}(id, b)}$

if $\mathrm{Valid}(id, b, \mathrm{BB}, \mathrm{pk})$ **then**
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Advantage of the adversary: $\left| \mathrm{P}\left[\mathrm{Exp}_{\mathcal{A}}^{\mathrm{priv},0}(\lambda) = 1\right] - \mathrm{P}\left[\mathrm{Exp}_{\mathcal{A}}^{\mathrm{priv},1}(\lambda) = 1\right] \right|$

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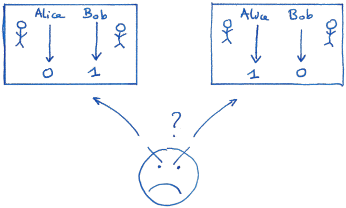

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Attacker has access to vote and cast oracles

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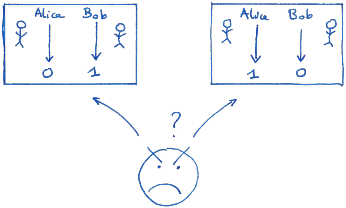

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$\underline{\mathcal{O}_{\mathrm{cast}}(id, b)}$

if $\mathrm{Valid}(id, b, \mathrm{BB}, \mathrm{pk})$ **then**
  $\mathrm{BB} \leftarrow \mathrm{BB} \| b$

Cast oracle:
cast b to the ballot box
for dishonest id

Advantage of the adversary: $\left| \mathrm{P}\left[\mathrm{Exp}_{\mathcal{A}}^{\mathrm{priv},0}(\lambda) = 1\right] - \mathrm{P}\left[\mathrm{Exp}_{\mathcal{A}}^{\mathrm{priv},1}(\lambda) = 1\right] \right|$

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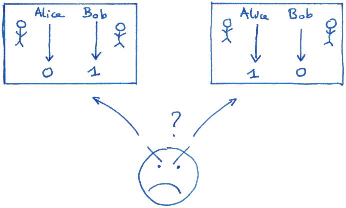

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Vote oracle:
choose two votes v_0, v_1
for honest voter *id*

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$\underline{\mathcal{O}_{\mathrm{cast}}(id, b)}$

v_β goes to the ballot box

v_0, v_1 are recorded

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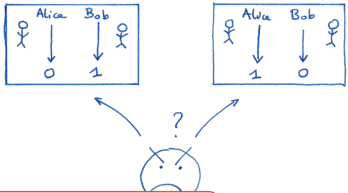

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Attacker sees the tally, guesses β

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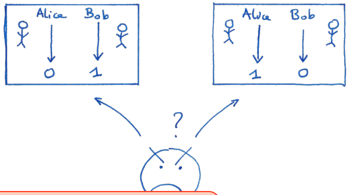

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Provided the distributions of votes for $\beta = 0, 1$ give the same result

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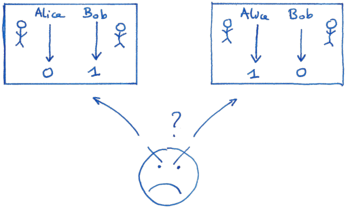

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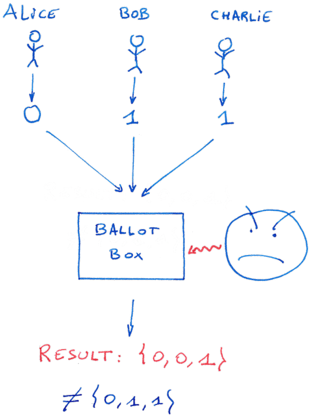
Individual verifiability: game-based definition



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$r \leftarrow \mathsf{Tally}(\mathsf{BB}, \mathsf{sk})$

if $\exists V_c.\ r = \rho(\mathsf{Voted} \cup V_c)$ **then**

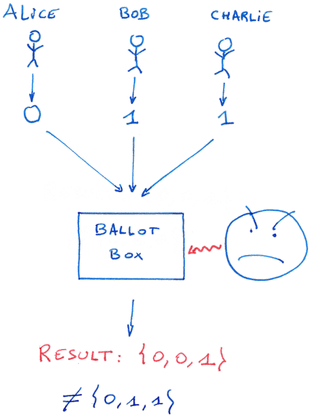
  return 0

else return 1

Advantage of the adversary: $\mathsf{P}\left[\mathsf{Exp}^{\mathsf{verif}}_{\mathcal{A}}(\lambda) = 1\right]$

Individual verifiability: game-b

As before:
Attacker has vote and cast oracles



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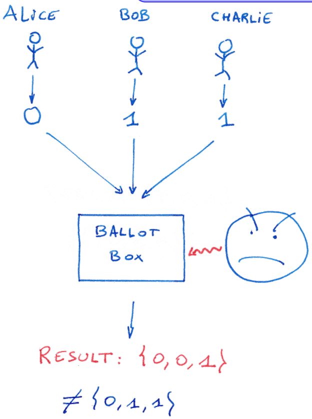

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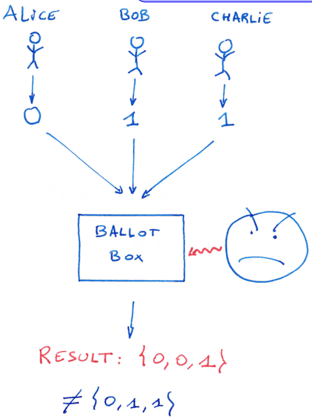
$\mathcal{O}_{\text{vote}}$: choose honest votes
$\rightarrow$ recorded in Voted



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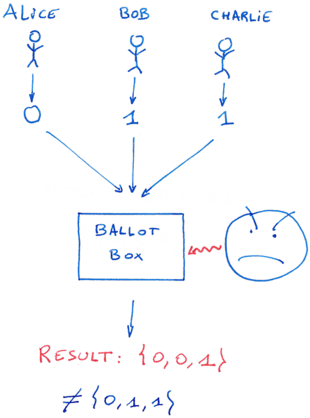

Compute the election result

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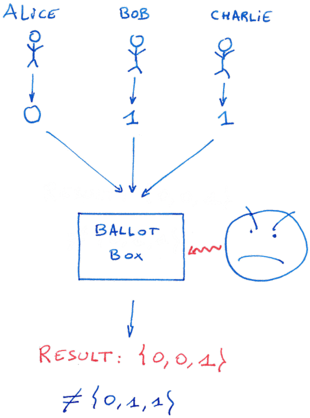

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Result contains at least honest votes?
if not: $\mathcal{A}$ wins

Advantage of the adversary: $\mathsf{P}\left[\mathsf{Exp}^{\mathsf{verif}}_{\mathcal{A}}(\lambda) = 1\right]$

Main result

Theorem (Privacy implies Individual Verifiability (computational))

$$\exists \mathcal{A}.\ \mathsf{P}\left[\mathsf{Exp}^{\mathsf{verif}}_{\mathcal{A}}(\lambda) = 1\right] \ \textit{not negligible} \quad \Longrightarrow$$

$$\exists \mathcal{B}.\ \left|\mathsf{P}\left[\mathsf{Exp}^{\mathsf{priv},0}_{\mathcal{B}}(\lambda) = 1\right] - \mathsf{P}\left[\mathsf{Exp}^{\mathsf{priv},1}_{\mathcal{B}}(\lambda) = 1\right]\right| \ \textit{not negligible.}$$

We also prove the same implication in a symbolic model (process algebra), to show its generality:

Theorem (Privacy implies Individual Verifiability (symbolic))

$$\forall \alpha, a, b.\ P_{\alpha \cup \{a \mapsto 0, b \mapsto 1\}} \approx P_{\alpha \cup \{a \mapsto 1, b \mapsto 0\}} \ \Longrightarrow$$

$$\forall \alpha.\ \forall (t.\mathtt{out}(ch_r, x), \phi) \in \mathsf{trace}(P_\alpha).\ \exists V_c.\ \phi(x) = \rho(\mathit{Voted}(t) \uplus V_c).$$

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Assuming there is an attack on individual verifiability,
we construct an attack on privacy.

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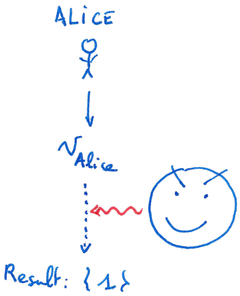

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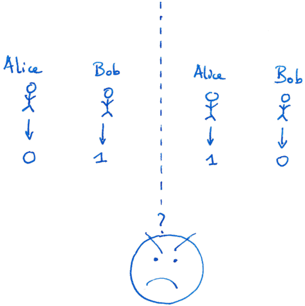
- consider an attacker against privacy



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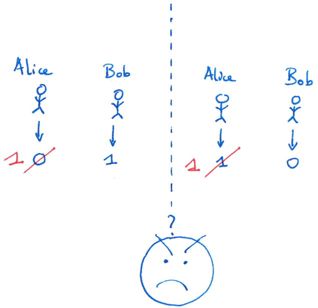

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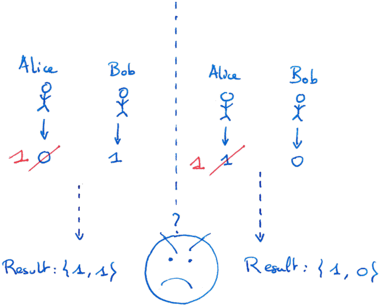
- the attacker turns Alice's vote to 1
- the result is $\{1, \text{Bob's vote}\}$



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- assume that
  the attacker can break verifiability
  by turning Alice's vote into 1
- consider an attacker against privacy
- the attacker turns Alice's vote to 1
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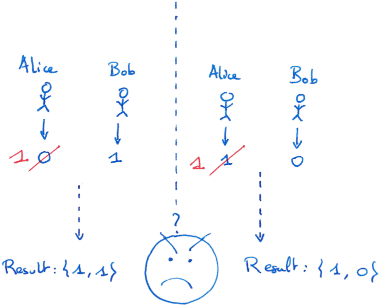

$\Longrightarrow$ the attacker learns Bob's vote, and breaks privacy

Intuition

Assuming there is an attack on individual verifiability,
we construct an attack on privacy.

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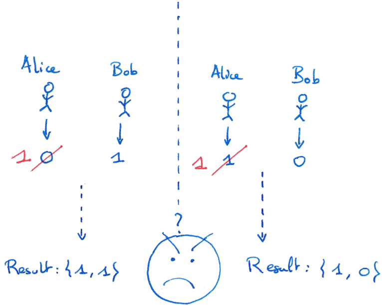

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We generalise this idea to any attack on verifiability.

Proof sketch (assuming a blank vote)

Assuming $\mathcal{A}$ breaks verifiability we build $\mathcal{B}$ that breaks privacy.

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$\beta = 0$ | $\beta = 1$

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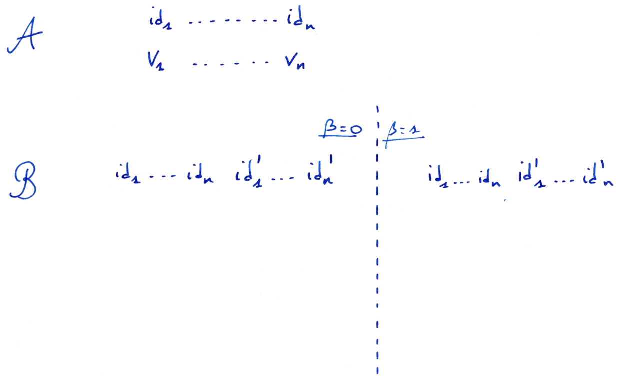

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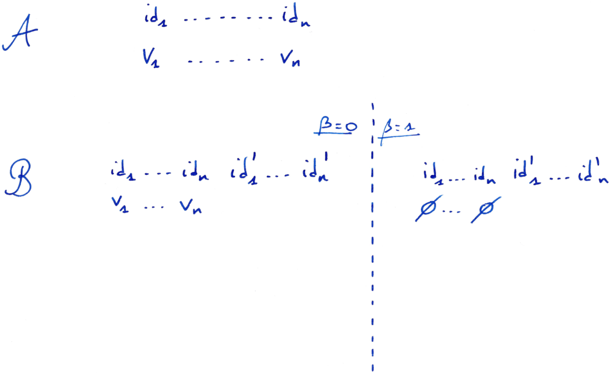

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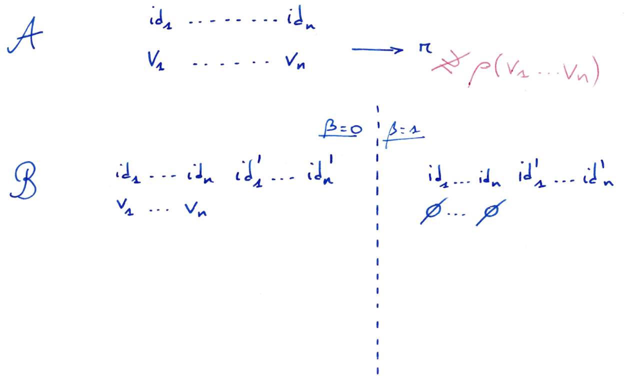

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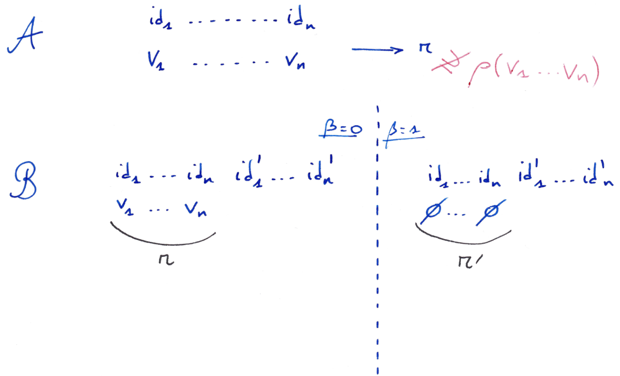

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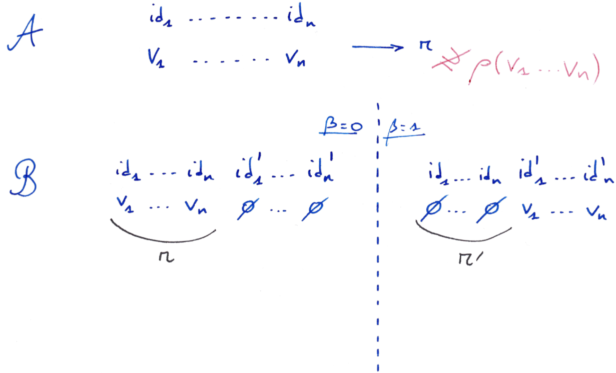

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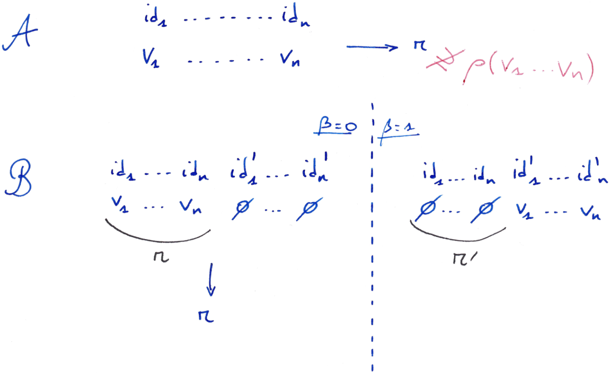

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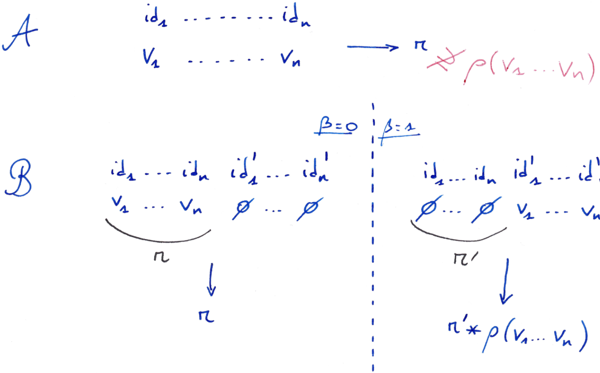

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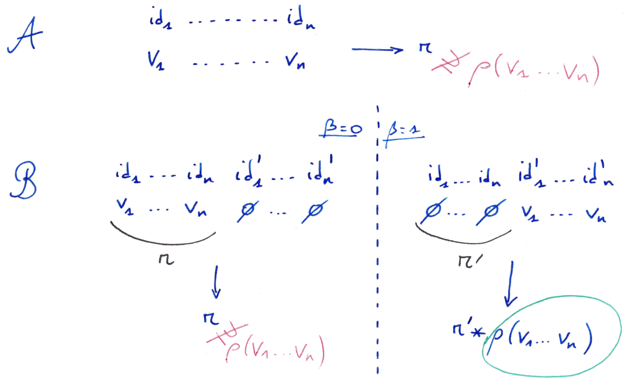

- $\mathcal{B}$ checks if the result contains all the v_i:
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  But protocols aim for privacy against a dishonest ballot box!

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- We need a new definition of privacy, against a dishonest ballot box

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if $\forall id \in \mathsf{V}_0, \mathsf{V}_1.\ id \in \mathsf{H} \ \wedge\ \rho(\mathsf{V}_0) = \rho(\mathsf{V}_1)$ **then**

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return $\mathcal{A}_3(\mathsf{pk}, r)$

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- We apply it to a few existing protocols, to show its relevance

Protocol	Honest box	Dishonest box naïve	Careful voters	
Helios	✓	✗	✗	[attack P. Roenne]
Belenios	✓	✗	✓	
Civitas (no revote)	✓	✗	✓	
Neuchâtel (no revote)	✓	✗	✗	[assumes an honest box]

✓: the protocol is private, ✗: attack on privacy

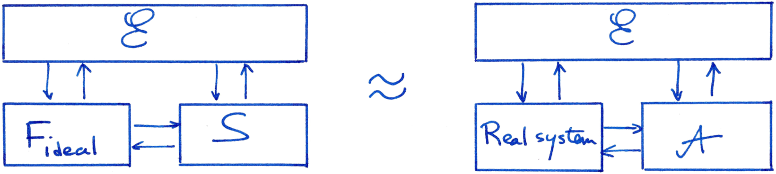
Work in progress: towards more precise definitions

- Privacy with careful voters is a first step, but not enough:
  only says something when everyone verifies
  = "among people who check, the attacker does not know who voted for who"

- Problem: not easy to have an indistinguishability game for voters who do not check
  = as soon as someone does not check, there *is* a loss of privacy

- Seems more doable with another way of writing properties

Simulation-based definition



- Idea: describe an ideal system, where the attacker "obviously" has no power
- Prove (reduction) that the ideal attacker can simulate everything the real one can do.

Ideal functionality for voting

Case of a honest ballot box:

Ideal functionality $F_{voting}(\rho)$ interacts with environment $\mathcal{E}$ and simulator S. $F_{voting}(\rho)$ accepts two kinds of messages:

- on input $vote(id, v)$ from $\mathcal{E}$ or S:
  store (id, v) in a list L, and send $ack(id)$ to S.
- on input *tally* from S, return $\rho(L)$ to $\mathcal{E}$ and S, then halt.

Clearly, S learns no information on the honest votes.

$\rightarrow$ Problem: with a dishonest ballot box, this cannot be realised
$\rightarrow$ Need to distinguish between voters who check and others

Conclusion

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$$\text{Privacy} \implies \text{Individual Verifiability}$$

- Proved in computational and symbolic models
- Better understanding of privacy: some verifiability is required!
- Highlights limitations of game-based current definitions:
  only honest ballot boxes [Bernhard, Smyth, 2014]
- A new definition of privacy against a dishonest ballot box
  $\longrightarrow$ modelling verification steps
- Limitation: assumes everyone checks their vote
  $\longrightarrow$ Future work: more plausible scenario where only some voters check